Dilatometry

Materials from the 2007 presentation of George Schmiedeshoff, Occidental College were heavily used
Dilation: $\Delta V$ (or $\Delta L$)

Intensive Parameters:

- **T**: thermal expansion: $\beta = \frac{d\ln(V)}{dT}$
- **H**: magnetostriction: $\lambda = \frac{\Delta L(H)}{L}$
- **P**: compressibility: $\kappa = \frac{d\ln(V)}{dP}$
- **E**: electrostriction: $\xi = \frac{\Delta L(E)}{L}$
- *etc.*
A (very) Brief History

- **Heron of Alexandria (0±100):** Fire heats air, air expands, opening temple doors (first practical application...).

- **Galileo (1600±7):** Gas thermometer.

- **Fahrenheit (1714):** Mercury-in-glass thermometer.

- **Mie (1903):** First microscopic model.

- **Grüneisen (1908):** $\beta(T)/C(T) \sim$ constant.
Phase Transition: TN

2nd Order Phase Transition, Ehrenfest Relation(s):

\[
\frac{dT_{N2}}{dp_c} = V_M T_{N2} \frac{\Delta \alpha_c}{\Delta C_p}
\]

1st Order Phase Transition, Clausius-Clapyeron Eq(s):

\[
\frac{dT_{N1}}{dp_c} = \frac{\Delta V}{\Delta S} \approx V_M \frac{\Delta (\frac{\Delta L}{L})}{\Delta S}
\]

\text{TbNi}_2\text{Ge}_2

(Ising Antiferromagnet)

Paul Ehrenfest (1880-1933)

PhD from Vienna Technical University
1904 – married Tatyana Alexeyevna Afanasieva – Russian mathematician
1907-1912 – St. Petersburg, Russia (happiest days of his life)
1912-1933 - professor in Leiden

Ehrenfest theorem
Ehrenfest paradox
Ehrenfest-Tolman effect
Classification of phase transitions
Ehrenfest time
Spinor

He was not merely the best teacher in our profession whom I have ever known; he was also passionately preoccupied with the development and destiny of men, especially his students. A. Einstein
Grüneisen Theory \textit{(one energy scale: }U_o\text{)}

\[
\Gamma \equiv -\frac{\partial \ln U_o}{\partial \ln V} = V_M \kappa(T) \frac{\beta(T)}{C_p(T)} = \text{const.}
\]

\[U_o \propto V^{-\Gamma}\]

e.g.: If \(U_o = E_F\) (ideal) then: \(\Gamma_{IFG} = \frac{2}{3}\)
**Grüneisen Theory** (multiple energy scales: $U_i$ each with $C_i$ and $\Gamma_i$)

\[
\Gamma \equiv \frac{\sum_i C_i(T) \Gamma_i}{\sum_i C_i(T)} = \Gamma(T)
\]

e.g.: phonon, electron, magnon, CEF, Kondo, RKKY, etc.

\[
\Gamma_{\text{eff}} \equiv V_M \kappa(T) \frac{\beta(T)}{C_p(T)} = \Gamma_{\text{eff}}(T)
\]

**Examples:**

Simple metals: \(\Gamma \sim 2\)

\[
\Gamma_e = \frac{2}{3} + \frac{d \ln(m^*)}{d \ln(V)}
\]
Example (Noble Metals):

After White & Collins, JLTP (1972).
($\Gamma_{\text{lattice}}$ shown.)
Example (Heavy Fermions):

$$\Gamma_{HF}(0)$$

After deVisser *et al.* (1990)
Dilatometers

- Mechanical (pushrod *etc.*).
- Optical (interferometer *etc.*).
- Electrical (Inductive, Capacitive, Strain Gauges).
- Diffraction (X-ray, neutron).
- Others (absolute & differential).
push-rod with inductive readout
(Ames Laboratory thermal analysis facility in Wilhelm Hall)
Optical dilatometer - interferometer

nanometer resolution, commercially available
Strain gauges

Gauge factor, \( GF = \frac{\Delta R}{R} / \frac{\Delta L}{L} \) is usually about 2 (for metal film gauges)
XRD dilatometry

Less accurate
Tedious/expensive
VERY useful for structural phase transitions (gives structural information)
Capacitive Dilatometer (Cartoon)

\[ C = \varepsilon_0 \frac{A}{D} \]
- Cell body: OHFC Cu.
- BeCu spring (c).
- Styecast 2850FT (h) and Kapton (i) insulation.
- Sample (d).
Calibration

- Use sample platform to push against lower capacitor plate.
- Rotate sample platform (θ), measure C.
- \( A_{\text{eff}} \) from slope (edge effects).
- \( A_{\text{eff}} = A_0 \) to about 1%?!
- “Ideal” capacitive geometry.
- Consistent with estimates.
- \( C_{\text{MAX}} \gg C \): no tilt correction.

Operating Region

\[ C_{\text{MAX}} \geq 65 \text{ pF} \]
Cell Effect

$$\alpha_{Sample} = \frac{1}{L} \left( \frac{dL}{dT} \right)_{Sample+Cell} - \frac{1}{L} \left( \frac{dL}{dT} \right)_{Cell+Cu} + \alpha_{Cu}$$

Much smaller cell effect, above ~10K, using quartz (or sapphire?) instead of Cu for cell body. See, for example, Neumeier *et al.* (2005) and references therein.
\[ \alpha \left(10^{-5} \text{ K}^{-1}\right) \]

- \(\text{Kroeger & Swenson (1977)}\)
- \(\text{this work}\)

\[ L = 3.15 \text{ mm} \]

**Aluminum**

Inset:

\[ |\Delta \alpha|_{\text{ave}} = 8.6 \times 10^{-8} \text{ K}^{-1} \]
Tilt Correction

- If the capacitor plates are truly parallel then \( C \rightarrow \infty \) as \( D \rightarrow 0 \).
- More realistically, if there is an angular misalignment, one can show that \( C \rightarrow C_{\text{MAX}} \) as \( D \rightarrow D_{\text{SHORT}} \) (plates touch) and that

\[
D = \frac{\varepsilon_0 A}{C} \left[ 1 + \left( \frac{C}{C_{\text{MAX}}} \right)^2 \right]
\]


- For our design, \( C_{\text{MAX}} = 100 \text{ pF} \) corresponds to an angular misalignment of about 0.1°.
- Tilt is not always bad: enhanced sensitivity is exploited in the design of Rotter et al. (1998).
Kapton Bad (thanks to A. deVisser and Cy Opeil)

- Replace Kapton washers with alumina.
- New cell effect scale.
- Investigating sapphire washers.
Torque Bad

- The dilatometer is sensitive to magnetic torque on the sample (induced moments, permanent moments, shape effects...).
- Manifests as irreproducible magnetostriction (for example).
- Best solution (so far...): glue sample to platform.
- Duco cement, GE varnish, N-grease...
- Low temperature only. Glue contributes above about 20 K.
Hysteresis ~Bad

- Cell is very sensitive to thermal gradients: thermal hysteresis. But slope is unaffected if T changes slowly.

- Magnetic torque on induced eddy-currents: magnetic hysteresis. But symmetric hysteresis averages to “zero”:

![Graph showing hysteresis](image)

\[(UP + DN)/2: \text{Slope} = -0.06 \, \text{A/T}\]
Quantum Oscillations

Can estimate uniaxial pressure derivatives of the extremal FS cross-sections

\[ \epsilon_i = -MH \frac{\partial \ln S_m}{\partial \sigma_i} \]
Tricritical Phenomena at the $\gamma \rightarrow \alpha$ Transition in Ce$_{0.9-x}$La$_x$Th$_{0.1}$ Alloys


[Graph and data analysis]
Anisotropic thermal expansion and magnetostriction of YNi$_2$B$_2$C single crystals

S L Bud'ko$^1$, G M Schmiedeshoff$^2$, G Lapertot$^{1,3}$ and P C Canfield$^1$
Magnetic-Field-Induced Lattice Anomaly inside the Superconducting State of CeCoIn$_5$: Anisotropic Evidence of the Possible Fulde-Ferrell-Larkin-Ovchinnikov State

V. F. Correa,$^{1,6}$ T. P. Murphy,$^1$ C. Martin,$^1$ K. M. Purcell,$^{1,2}$ E. C. Palm,$^1$ G. M. Schmiedeshoff,$^3$ J. C. Cooley,$^4$ and S. W. Tozer$^1$

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(a)

![Graph showing $\Delta L_c/L_c$ versus $B$ for different temperatures.]

(b)

![Graph showing $\lambda(T)$ versus $B$ for different temperatures.]

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Extreme Conditions Group
The Good & The Bad

- Small (scale up or down).
- Open architecture.
- Rotate *in-situ* (NHMFL/TLH).
- Vacuum, gas, or liquid (magnetostriction only?).
- Sub-angstrom precision.

- Cell effect ($T \geq 2K$).
- Magnetic torque effects.
- Thermal and magnetic hysteresis.
- Thermal contact to sample in vacuum ($T \leq 100mK$).
Recommended:

- **Book (broad range of data on technical materials etc.):** *Experimental Techniques for Low Temperature Measurements*, Ekin, 2006.
- **Book:** *Thermal Expansion*, Yates, 1972.
- ...and references therein.